KATHERINE SOCHA'S EDITED VERSION

What is algebra and why do students find it so hard?

Algebra is not just arithmetic with letters standing for numbers. Doing algebra requires a different kind of thinking.

Many people find arithmetic hard to learn, but most succeed, though only after a ***lot*** of practice. Learning arithmetic is possible because the basic building blocks of arithmetic, numbers, arise naturally in the world, when we count things, measure things, buy things, make things, use the telephone, go to the bank, check the baseball scores, etc. Numbers may be abstract — you never saw, felt, heard, or smelled the number 3 — but they are tied closely to all the concrete things in the world we live in.

In algebra you are a second step of abstraction removed from the everyday world: those x's and y's usually denote numbers ***in general***, not particular numbers. In algebra you use analytic, ***qualitative*** reasoning (thinking logically) **about** numbers, whereas in arithmetic you use numerical, ***quantitative*** reasoning (computing explicitly) ***with*** numbers.

For example, you need to use algebraic thinking to program a spreadsheet like Microsoft *Excel* to repeat a lot of calculations. It doesn't matter whether the spreadsheet is for calculating scores in a sporting competition, keeping track of your finances, running a business, or figuring out the best way to equip your character in *World of Warcraft*: you need to think algebraically to set it up to do what you want — that means thinking about or across numbers, rather than in terms of numbers. In contrast, you need to use arithmetic thinking to calculate the percentage of successful free throws your kid made in last night's game or the total cost of your groceries with sales tax.

When students start to learn algebra, they inevitably try to solve problems by arithmetical thinking. That's natural, given all the effort they have put into mastering arithmetic. And at first, when the algebra problems they meet are particularly simple (that's the teacher's classification), this approach works. In fact, the stronger a student is at arithmetic, the further they can progress in algebra using arithmetical thinking. (Many students can solve the quadratic equation $x^2 = 2x + 15$ using only basic arithmetic, with no algebra at all.) Paradoxically, or so it may seem, those better students may find it harder to learn algebra. Because to do algebra, you have to ***stop*** thinking arithmetically and start thinking algebraically. Arithmetic makes the notes; algebra makes the song.

[Notes from KS edits: the ending seems to lack a little oomph. I suggest adding a colorful analogy with the take home lesson, such as the final sentence above. The author may over-ride this. I also felt the fourth paragraph needed the

counterpoint of a sentence contrasting arithmetic thinking with algebraic thinking. I added a suggested sentence. The author may over-ride. Word count (not including title): 394.]