JANET BEERY'S EDITED VERSION

What is algebra and why do students find it so hard?

Algebra is not just arithmetic with letters standing for numbers. It is a different kind of thinking. But there is a wide and well-traveled path from arithmetic up to algebra that most everyone can climb with enough time and effort.

Many people find arithmetic hard to learn, but most succeed, though only after a ***lot*** of practice. What makes mastery possible is that the basic building blocks of arithmetic, numbers, arise naturally in the world around us, when we count, measure, buy, and make things, use the telephone, go to the bank, check the baseball scores, etc. Numbers may be abstract — you never saw, felt, heard, or smelled the number 3 — but they are tied closely to all the concrete things in the world we live in.

Algebra is a second step of abstraction removed from the everyday world: those x's and y's denote numbers ***in general***, not particular numbers. In algebra you use analytic, ***qualitative*** reasoning ***about*** numbers, whereas in arithmetic you use numerical, ***quantitative*** reasoning ***with*** numbers. Algebra is thinking ***logically*** about numbers; arithmetic is computing with numbers.

But there is plenty of algebra in the world around us. When you compute the square footage of your new herb garden as Area = Length x Width (or A = L x W), or the price of a 20% off sale item as Sale Price = Regular Price – 2 x 1/10 x Regular Price, or your gas mileage for a road trip as Gas mileage = Distance traveled / Gas used, you're using algebraic formulas. When you determine that an herb garden with fixed area 6 square feet and fixed length 4 feet must have Width = 6/4 = 1.5 feet or that, in general, Width = Area / Length, you are doing algebra. Likewise, when you set up a spreadsheet to compute taxes for your business or batting averages for your baseball league, you are doing algebra. (Your algebra makes the computer do arithmetic for you!)

Learning algebra does not require a sudden miraculous leap but rather a steady deliberate climb from solving the puzzle 4 + ? = 7 (at a very young age) to solving for t (time) in the formula $B = P(1 + 0.25r)^t$ for the amount of money in your bank account. The climb from arithmetic to algebra may be steep in places. But reaching the pinnacle means you possess the powerful tool that makes the modern world go 'round.

Note from JB, editor of *Big City Times* opinion pages, to KD: I appreciate your effort to educate and reassure parents and community members about the nature and difficulty of algebra, written in your usual lively and inimitable style. However, my sense is that your piece, as written, may frighten parents into thinking that they and their children could never learn a subject as abstract as algebra, and I do not believe this was your intent. Assuming you believe most or all students can learn algebra and would like to communicate this message, I would like to suggest changes such as those I have made above that attempt to provide more of a link between arithmetic and algebra (and for which I owe thanks to my neighbor, a prescient and precocious 7th grader at Big City Middle School, and her algebra teacher there). Another alternative would be to maintain your insistence on the distinction between the two, but make algebra seem more tractable. Could you please try to make algebra seem a little less abstract and scary?

Obviously, I'm not an expert in this area but I am interested in finding a way to run the piece. I took the liberty of consulting my alter-ego, also named JB, who claims to be trained as an "algebraist", but she wasn't much help. She said something about giving an example of solving an equation by arithmetic guessing vs. algebraic "undoing" and she wanted you to give an example of an actual spreadsheet formula. I told her that, if we end up using your piece in print form, we could include additional examples in a more detailed version at the website. Fortunately, I have a friend, MN, who is an expert in the teaching and learning of algebra and who promises to send you suggestions for your piece by next Monday….

KD's COMMENT: This is where doing this in a "he says, she says" fashion by email — restricted to one shot each — falls way short of a conversational exchange, which is what I would embark on next. The editor knows her audience and clearly wants to run my piece. Unfortunately, I think her first suggested alternative text goes too far and the result is not at all what I would want to say. The initial examples she introduces are, I would say, not at all algebra, but arithmetic. (Plugging numbers into a formula is pure arithmetic and does not involve any arithmetic thinking. The editor chooses her words very carefully here, and respects the distinction, but I worry the damage is already done, and false beliefs have been reinforced.) On the other hand, those kinds of example are likely to be familiar (as examples of algebra) to many if not most readers of the newspaper, since that is what teachers spend a lot of time doing in the "Algebra Class".

Unfortunately, in the US school system, much of what goes under the name "Algebra" is not algebra at all but arithmetic of the real number system. I would not want the reader to get the impression that algebra is just "arithmetic with symbols". Now the editor's confidant, the mysterious JB, clearly knows the distinction. Her suggestion is to make the transition from arithmetic to algebra seem less of a chasm and more of a climb, albeit one that may be "steep in places." My task now as the author of the Op-Ed is to satisfy the editor's desire to connect to her readers with my desire to be honest about the nature of algebra. I think it is important to be open about the fact that doing algebra is fundamentally different from doing arithmetic.

[As MAA members doubtless are aware, opinions differ — sometimes hotly so — as to whether the transition from arithmetic to algebra involves an unavoidable discontinuity. I happen to think that, for all practical purposes, there is a discontinuity. But a newspaper Op-Ed is not the place to get involved in that issue. So I'll keep that to myself.]

Seeking convergence with the editor on a middle ground would be a fascinating — and for me familiar and invariably enjoyable — experience. Real editors rarely have a mathematically-expert confidant on call, so they have to rely on the mathematician author to provide the expertise. But they can be assumed to know their readers. I may not want to use the editor's opening examples, at least not without some qualifying comments, but her point seems a valid one: I do need to connect to the readers. If I can't find a way to meet the editor's demands, then my article won't be published. Ask yourself who is the loser.